How To Normalize Financial Markets ?

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There exists a monetary policy that "normalizes" stock returns by targeting the nominal short-term interest rate to eventually equal its real expectation of 1.8%, as inflation expectations are targeted to be zero in the long run. This monetary policy, characterized as the Wicksell rule, can be proven to guide the stock market to a rational expectations equilibrium, such that real returns quickly and smoothly converge to a normal distribution. Interestingly, the bounded variance of real stock returns is asymptotically less than the variance of the economy's random shocks of news. Proving this requires that a nonlinear, stochastic model be constructed to simulate the behavior of the stock and money markets as part of the possible evolutions of the economy's forecasting errors. The research of many economists can be integrated in a novel way to build this dynamic macro model of a complex learning economy, consistent with a Keynesian framework.

Field of Research: Monetary Policy & Financial Forecasting

1. Introduction

The Federal Reserve System, the U.S. central bank, currently targets the federal funds rate that controls interest rates to stabilize the economy. According to Taylor (1993), it generally raises (lowers) the short-term, nominal interest rate, when inflation expectations are too high (low) and real output is greater (lower) than its trend level. This Taylor rule is based on a misspecified model of stabilizing the economy, which causes a central bank to overreact to changes in inflation and output. Implementing this policy will cause financial markets to behave chaotically like a Rössler system, a very simple and elegant way to model chaos in continuous time (Rössler, 1976, Haley, 2006).

If the Federal Reserve actually followed the advice of Taylor (2006) to contain inflation in 2007, the federal funds rate would have increased to 6.5%. But this policy, which he proposed as the undersecretary of the U.S. Treasury, would have made the economy worse off than it is today. Interestingly, the Fed raised the federal funds rate to precisely the same target in 2000 to restrain the economy that was then over-expanding. This precipitated the collapse of the biggest stock market bubble in American history, contributing to a U.S. recession in 2001.

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Even though the Fed avoided making this mistake, it still kept interest rates too high for too long a time, causing a precipitous slow-down of the economy that is now in recession. Worse, is the Fed making money too cheap, cutting the federal fund rate to less than .25% from its previous high of 5.25% to stimulate the economy? Does it risk distorting the allocation of capital, eventually creating a speculative bubble as the economy recovers? This overreaction based on biased forecasts has occurred before, such as during the speculative boom-bust cycle of the Nasdaq stocks in 1998-2001 and the current collapse of America's real estate bubble. Thus it is not evident at all, whether the Fed knows how to prevent this irrational exhuberance from emerging again. Economists have so far failed to adequately explain the dynamics of how the economy learns from its mistakes. Muth (1961) claims that economic expectations are rational, such that forecasts are unbiased since expected errors are always zero. According to Guesnerie and Woodford (1995):

"Contrary to Muth's claim ... (that) the rational expectations hypothesis is nothing else than the extension of the rationality hypothesis to expectations, we need a theory why the rational expectations equilibrium is reached..."

This failure explains why economists, including the Fed, cannot make accurate forecasts. It is now possible to theoretically meet the challenge of developing a more reliable dynamic model of the financial markets as part of the evolution of a complex learning economy. The research of many economists can be integrated in a novel way to build this model. For example the observations of Nelson (1996) and Diebold & Rudebusch (1999) can be modeled to explain how the economy chaotically fluctuates due to nonlinear feedback.

The even more complex behavior of the stock market can be modeled to be the source of chaotic fluctuations of risky returns, causing excess kurtosis. Though under the right circumstances real stock returns mean-revert to a normal density of returns. A nonlinear differential equation of real stock returns has been previously developed by Haley (2001, 2006) that incorporates the relevant research of Scheinkman & LeBaron (1989), Engle & Lee (1996), Campbell, Lo & MacKinlay (1997), and Siegel (2008). This complexity also affects the money market, since increasing (decreasing) stock returns more likely raise (lower) the federal funds rate by the Fed (Rigobon & Sack, 2003).

If a central bank targets interest rates by overreacting to changes in inflation and output, chaos can suddenly emerge in the financial markets. For example, available liquidity in the money market could be disrupted by interest rates being biased too high or low. Then financial markets can behave like a Rössler nonlinear model of chaotic forecasting errors of real stock returns, nominal interest rates, and inflation. Much of this paper's analysis of the financial market's nonlinear dynamics was first presented at the 2006 World Business Institute Conference in Melbourne, Australia. Based on a Rössler financial forecasting model, a forecast of increasing financial volatility was discussed at the January 2007 American Mathematical

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Society (AMS) Meetings in New Orleans, Louisiana. At that time based on the forecasting models that are analyzed in this paper, an economic slowdown was predicted, because interest rates were too high. The failure to cut interest rates in a timely fashion now has led to a major recession that almost all economists failed to forecast. But it can be proven that there exists a monetary policy first described by Wicksell (1898), which guides the economy's search for a rational expectations equilibrium. Specifically, a real interest rate peg can make the forecast errors of real stock returns behave like a Langevin error correcting, differential equation, which smoothly and quickly converges to a normal density with with a bounded variance. Furthermore this variance is less than the variance of random shocks from news experienced by the economy.

2. A Complex Learning Economy

The system of differential equations, summarized in Table 1, explains how a complex economy learns to correct its forecast and coordination errors. This search evolves in continuous time, making the analysis more tractable. Once a central bank selects its monetary policy, the dynamics of economic errors implies a specific economic model or monetary regime, which predicts the actual levels of real output, real stock returns, nominal interest rates, inflation, and the money supply. Furthermore, these possible dynamic models are consistent with a Keynesian framework (See Appendix).

Under different monetary regimes, it is then possible that a learning economy's evolution persistently deviates from its trend. The process is complex, because the dynamics occur in five dimensions with many interactions. The complexity becomes even greater, when nonlinear feedback exists in the financial markets, making the evolution of stock market bubbles possible. For simplicity, the following analysis assumes that the economy is closed to international trade; expectations are constant; there is no deficit spending by the government; and tax rates are fixed. Also all parameters are assumed to be positive constants.

Assume that real output for a closed economy, Q, generally mean-reverts to its trend as it dynamically adjusts to excess stock returns, Z, the difference between risky real stock returns, R, and the real short-term, risk-free interest rate, r. Furthermore, assume that real output rises (falls), as the forecast error, Π, for excess real returns, Z, increases (decreases) with respect to its fixed expectation, z_e. The following stochastic linear differential equation explains this evolution of the coordination errors, ϕ, of the natural logarithm of the economy's real output, Q, with respect to its expectation, which for simplicity is assumed to be its trend level, Q_T, that grows at a constant rate:

$$\dot{\phi} = \hat{Q} = \alpha_1 \Pi - \alpha_2 \phi, \text{ if } \dot{\hat{Q}}_T = 0, \tag{A1}$$

such that $\phi = \ln Q - \ln Q_T$, $Z = R - r$, $\Pi = Z - z_e$, $\hat{Q} = \dot{Q}/Q$, $\dot{Q} = \dfrac{dQ}{dt}$.

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It easily follows that there is a negative slope of the stationary locus of (A1) or what the Keynesians call an IS curve (See (L1) in Appendix).

The interaction of random noise with nonlinear feedback between the money and stock markets can increase the likelihood of excess kurtosis of extreme stock returns that emerges during financial bubbles and panics (Engle & Lee, 1996). This complex speculative behavior is described in the following nonlinear stochastic differential equation that is perturbed by noise, ε_1, caused by exogenous random shocks of news, which is normally distributed with a zero mean and a standard deviation of σ_1. The followinig assumption specifies the dynamic mean-reverting behavior of real stock returns, R, that nonlinearly interact with the forecast errors, x^*, of the nominal, short-term interest rate, r^*, with respect to its fixed expectation, r_e^*:

$$\dot{z} = \dot{R} = \beta_0 - \beta_1 x^* z - \beta_2 z + \varepsilon_1, \text{ if } \dot{r}_e = 0, \ \dot{z}_e = 0, \tag{A2}$$

such that $z = R - r_e$, $z_e = R_e - r_e = \dfrac{\beta_0}{\beta_2} > 0$, $x^* = r^* - r_e^*$, $\varepsilon_1 \sim N(0, \sigma_1)$,

and β_2 is sufficiently large to ensure fast convergence.

It is important to note under the right circumstances, that expected excess real stock returns, z, with respect to the expectation of the real rate of interest, r_e, can quickly converge to its expectation, z_e, the market risk premium. This occurs when the short-term nominal interest rate, r^*, equals its expectation, r_e^*, as long as β_2 is a large positive number. In this case real stock returns are normally distributed in the long run, since the above differential equation, which is perturbed by noise, ε_1, coming from the random shocks of news, reduces to a mean-reverting Langevin equation.

The nominal short-term interest rate of risk-free government bonds increases (decreases), if the forecast error, π^*, for excess nominal stock returns, z^*, with respect to the expectation of the nominal interest rate, r_e^*, are positive (negative) and the excess demand for real money, θ, is positive (negative). Then the evolution of the forecast errors, x^*, of the nominal, short-term interest rate, r^*, with respect to its fixed expectation, r_e^*, is given by the following linear differential equation:

$$\dot{x}^* = \dot{r}^* = a_1 \pi^* + a_2 \theta, \text{ if } \dot{r}_e^* = 0, \tag{A3}$$

such that $x^* = r^* - r_e^*$, $\pi^* = z^* - z_e^*$, $r_e^* = r_e + \hat{p}_e$, $Z_e^* = R_e^* - r_e^*$.

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Interestingly, if a_1 in the above differential equation is zero, assumption (A3) resembles the standard Keynesian dynamics of the money market, depending on how excess demand for real money, θ, is specified (Ferguson & Lim, 1998). In fact certain monetary policies can make this differential equation behave in a mean-reverting manner, such that there is a positive slope of what Keynesians call the LM curve (See (L2) in Appendix). But financial chaos can emerge in the money market, when a_1 is positive, because the nonlinear dynamics of the stock market can effect the money market. And the research of Rigobon and Sack (2003) observes that this interaction is possible.

Assume that the inflation rate, $\hat{p}$, accelerates (decelerates) when the short-term, nominal interest rate, r^*, is less (greater) than its expectation. Also assume that inflation increases (decreases), when excess demand for money, θ, and forecast errors of inflation, y, are both positive (negative). The following linear differential equation explains these dynamics, assuming for simplicity that the expectation of inflation, $\hat{p}_e$, is constant:

$$\dot{y} = \dot{\hat{p}} = -b_1\, x^* + b_2\, y + b_3\, \theta, \text{ if } \dot{\hat{p}}_e = 0, \tag{A4}$$

such that $y = \hat{p} - \hat{p}_e$.

This equation implies that the forecast errors for inflation can cycle in complex ways, such that inflation or deflation can distort forecasts. For example, under the right conditions it can be shown that a traditional Phillips curve exists, such that there is generally a positive trade-off between inflation and economic output. However, if expected economic errors become zero, the Phillips curve must be vertical, such that forecast errors of inflation do not affect expected output in the long run (See (L3) in Appendix).

To complete this dynamic system of error correcting equations an assumption must be made about the excess demand for real money, θ, that is defined as the demand for real money less than the supply of real money, M / p, both measured as natural logarithms. So assume that demand for the natural log of real money is inversely related to the short-term, nominal interest rate, r^*, which equals the real risk free, short-term rate of interest, r, plus the rate of inflation, $\hat{p}$. Also assume that real demand for money varies directly with the natural log of real output, Q. So excess demand for money behaves as follows:

$$\theta = -\, l_1\, r^* + l_2 \ln Q - \ln \frac{M}{p}, \text{ if } r^* = r + \hat{p}. \tag{A5}$$

Now it is possible to restate the definition of the excess demand for real money as varying directly with the coordination error, ϕ, of the log of real output and inversely

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related to the forecast error, x^*, of the nominal interest rate and excess real liquidity, L. Then Keynes' concept of transaction and speculative demand for money becomes clearer (See (L4) in Appendix).

3. How Policy Makes Markets Rational or Not

The evolution of a complex economy's error correcting system, summarized in Tables 1 & 2, is best understood by analyzing the dynamics holistically. One way to see the system's complexity is to discover how different monetary policies affect the economy's process of learning from its mistakes. By experimenting with alternative policy rules it can be proven that it is possible to efficiently speed up the convergence to an unbiased stochastic steady state of forecasting and coordination errors.

An approach that should simplify the proof of the existence of this rational expectations equilibrium is to reduce the uncertainty that distorts and disrupts the economy. One source of noise, resulting from random news, specified by ε_1 in Assumption (A2), cannot be controlled, since the variance of this noise is exogenous and is not effected by monetary policy. But increased financial instability may be caused by the expectation of inflation or deflation that disrupts the economy, according to Meltzer (1986). This is why Taylor (2006) and other economists believe that maintaining long-term price stability reduces volatility in the economy.

The benefits of this policy become evident by assuming that the standard deviation of inflation expectations, σ_2, is zero, if the expectation of inflation, $\hat{p}_e$, is also zero, This assumption reduces the uncertainty facing the economy as follows:

$$\hat{p}_e = 0 \text{ if and only if } \sigma_2 = 0. \tag{A6}$$

For simplicity, assume that the expectation of the real money supply is M_T / p_e, which is the trend level of the nominal money supply, M_T, divided by the expectation of the price level, p_e. Furthermore it can be assumed that the log of the trend real money supply equals the log of the demand for money that varies inversely with the expectation of the nominal interest rate, r_e^*, and directly varies with the log of the economy's trend real output, Q_T, as follows:

$$\ln \frac{M_T}{p_e} = -l_1\, r_e^* + l_2\, \ln Q_T. \tag{A7}$$

Then from assumptions (A5), (A6) and (A7), it is possible to target inflation expectations by targeting the money supply in the long run. Thus the growth rate of

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the trend money supply, $\hat{M}_T$, varies directly with the growth rate of the economy's trend real output of the economy, $\hat{Q}_T$. This means that the noninflationary growth rate of the trend money supply is targeted to be:

$$\hat{M}_T = l_2 \, \hat{Q}_T, \text{ if and only if } \hat{p}_e = 0, \, \dot{r}_e = 0. \qquad (T1)$$

Since l_2 is approximately one, the expectation of price stability requires that the average growth of money should be in an estimated range of 3-3.5%, based on past trends of the long run growth of real output.

So far, the model specified by assumptions (A1) through (A7) is complete in terms of describing the evolution of economic errors. If the objective is to predict the behavior of output, real stock returns, interest rates, and inflation, a properly specified monetary policy is required to completely describe possible alternative monetary regimes of a complex learning economy. Knowing how the interest rate as well as the money supply are targeted completes the dynamic system, representing the economy's evolution. Consequently, sufficient central bank intervention is essential to avoid indeterminancy in the model (Cass, 1995).

It can be proven that there exists a policy, which approximates how the Fed now targets interest rates to vary. The policy requires targeting the short-term nominal interest rate, r^*, to vary directly with the sum of its weighted adjusted real expectation, $r_e - w_1 z_e$, plus the expectation of inflation, $\hat{p}_e$, and the weighted coordination error, ϕ, for the natural log of real economic output, Q, as r^* varies inversely with a weighted excess real liquidity, L. This monetary policy is mathematically specified as follows:

$$r^* = (r_e - w_1 \, z_e) + \hat{p}_e + w_2 \, \phi - w_3 \, L,$$

$$\text{if } \quad w_1 = \frac{a_1}{a_2 l_1}, \, w_2 = \frac{l_2}{l_1}, \, w_3 = \frac{1}{l_1}, \qquad (A8)$$

such that $r^* = r + \hat{p}$, $r_e^* = r_e + \hat{p}_e$, $\phi = \ln Q - \ln Q_T$, $L = \ln \dfrac{M}{p} - \ln \dfrac{M_T}{p_e}$.

The policy rule implies that if inflation expectations, $\hat{p}_e$, increases (decreases) and real output, Q, is greater (less) than its trend, the target interest rate should rise (fall), assuming its real adjusted expectation, $r_e - w_1 z_e$, and excess real liquidity, L, are fixed. This is consistent with the Taylor rule, which constrains the economy when real output and the expectation of inflation are too high by raising interest rates, or it stimulates the economy when real output and inflation expectations are too low by lowering interest rates (Taylor, 1993). Furthermore this policy rule is equivalent to the following excess demand for real money:

$$\theta = \frac{a_1}{a_2} \, z_e \, .$$

Applying this rule to a complex learning economy given by the differential system (A1) through (A7), a monetary regime then emerges. After making the necessary substitutions in assumption (A3), it follows that:

$$\dot{x}^* = a_1 \, \pi^* + a_2 \frac{a_1}{a_2} \, z_e = a_1 (z + y),$$

since by definition, $\pi^* = z^* - z_e^* = z + \hat{p} - (z_e + p_e) = \pi + y$.

Clearly, after substituting again for θ in assumption (A4) it can be shown:

$$\dot{y} = -b_1 \, x^* + b_3 \frac{a_1}{a_2} \, z_e + b_2 \, y.$$

Combining these two differential equations with assumption (A2), it can now be proven that the dynamic model of the financial markets unperturbed by noise converges to a strange attractor. So from assumption (A1) output chaotically cycles around the economy's trend level of output, never converging to equilibrium, as long as excess stock returns are chaotic as well. According to Haley (2006 & 2007), this can be proven after making the necessary substitutions with the proper scaling for $\tilde{x}$ and $\tilde{\beta}$.

Consequently, the following three theorems represent a Rössler system of three differential equations, which follow from assumptions (A2), (A3) and (A4), assuming a Taylor-like policy regime specified by assumption (A8):

$$\dot{z} = \beta_0 + \tilde{x} \, z - \tilde{\beta} \, z , \qquad\qquad \text{(T2)}$$

$$\text{if } \tilde{x} = - b_1 \, x^* + b_3 \frac{a_1}{a_2} \, z_e , \ \beta_1 = b_1 , \ \tilde{\beta} = \frac{b_3}{b_1} \frac{a_1}{a_2} \, z_e + \beta_2 ;$$

$$\dot{\tilde{x}} = - (z + y), \qquad\qquad \text{(T3)}$$

$$\text{if } \dot{\tilde{x}} = - b_1 \, \dot{x}^* , \ a_1 \, b_1 = 1 ;$$

$$\dot{y} = \tilde{x} + b_2 \, y , \qquad\qquad \text{(T4)}$$

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$$\text{if } \tilde{x} = -b_1 x^* + b_3 \frac{a_1}{a_2} z_e .$$

Specifically, if $b_2 = \beta_0 = 0.2$ and $\tilde{\beta} = 5.7$, the above unperturbed system, embedded in the model of the economy, becomes chaotic according to Rössler (1976). It is a simple and elegant way to model chaos in continuous time. Also these parameter values that have economic meaning imply that the risk premium for stocks equals the ratio of β_0 divided by $\tilde{\beta}$, which in this case is approximately 4%. Thus the following proposition about the macroeconomy can now be proven from assumptions (A1) through (A8):

Financial markets "bubble" like a Rössler system perturbed by noise, (T5) assuming that interest rates are targeted by a Taylor-like rule, which causes the whole economy to cycle aperiodically. Even though the economy seeks to correct its forecasting and coordination errors, it fails because it over-reacts to changes in inflation and real output. Therefore as an economy recovers from recession it can overshoot its trend level of real output, because there is confusion about the economy's direction due to misspecified guidance by the central bank. Or as Keynes (1936) explained it:

"The latter stages of the boom are characterized by... conditions, which are unstable and cannot endure. A boom is a situation in which over-optimism triumphs... (that) in a cooler light, would be seen as excessive."

This "irrational exhuberance" described by Shiller (2000) emerges in the stock market as the economy over-expands. The resulting speculation can only be corrected when the short-term nominal interest rate is properly targeted to reduce economic uncertainty by eliminating financial chaos.

4. A Better Way to Stabilize the Stock Market

Whether expectations become rational critically depends on the stock market being efficient, such that it quickly converges to a stochastic equilibrium. Then expected excess returns of a diversified portfolio of stocks equal the market risk premium, and the probability density of excess returns is asymptotically normal. The stock market only avoids the financial chaos, caused by nonlinear feedback from the money market, when financial markets become synchronized, a necessary condition for rational expectations to emerge. Thus the variance from random supply shocks is reduced asymptotically by reinforcing mean-reversion as the economy learns to correct its errors. Thus, one condition for a stabilization policy to exist that dampens economic fluctuations is to peg the nominal interest rate, r^*, to equal its expectation, r_e^*, as follows:

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$$r^* = r_e^* .\qquad\qquad\qquad (A8)^*$$

Now it is easily follows from assumptions (A6) and (A8)* that the interest rate peg should equal the real rate expectation, r_e, if price stability is the other necessary condition for further reducing economic uncertainty:

$$r^* = r_e + \hat{p}_e = r_e, \text{ if and only if } \hat{p}_e = 0 .\qquad\qquad (T6)$$

This Wicksell rule combined with assumptions (A3) and (A5) implies the following behavior of the money market, measured by excess real liquidity, L, which varies directly witht the forecast errors of real output, Q, and excess nominal stock returns, z^*, with respect to the expectation of the nominal rate of interest, r_e^*:

$$L = l_2 \, \phi + \frac{a_1}{a_2} \, \pi^*, \text{ if and only if } x^* = r^* - r_e^* = 0 ,$$

such that $\phi = \ln Q - \ln Q_T$, $\pi^* = z^* - z_e^* = \pi + y$.

Then it can be shown that the complex learning economy becomes bifurcated and reduces to a self-correcting monetary regime, after substituting the above values of x^* and θ into the appropriate error-correcting differential equations in Table 1. So a different monetary regime emerges by replacing assumption (A8) with (A8)*.

A new derived system of differential equations that are perturbed by noise can be solved. Specifically, after substituting assumption (A8)* into (A2), the following error correcting, Langevin equation can be derived for the forecast errors, π, of real stock returns, R, with respect to its expectation, R_e, shocked by noise, ε_1:

$$\dot{\pi} = - \beta_2 \, \pi + \varepsilon_1, \text{ if } \pi = z - z_e, \ x^* = 0, \ z_e = \frac{\beta_0}{\beta_2}, \ \varepsilon_1 \sim N(0, \sigma_1) \quad (T7)$$

Furthermore, the long run stochastic equilibrium of this mean-reverting differential equation can be proven to be a normal density of π with a zero mean and standard deviation of σ_3, which is given by:

$$\text{Asymptotically, } \pi \sim N(0, \sigma_3), \text{ if } x^* = 0,\qquad\qquad (T8)$$

such that $\lim_{t\to\infty} E(\pi) = 0$, $\lim_{t\to\infty} \sigma_3^2 = \sigma_1^2 / 2\beta_2$, $\pi = R - R_e$.

Eventually the long-term variance of the unbiased forecast errors of real stock returns, σ_3^2, is less than the variance of the continuous random shocks of news, σ_1^2, assuming that β_2 is relatively large (Lasota & Mackey, 1994). Furthermore under certain parameter specifications it can be proven that the whole economy

becomes more rational, since expected forecast and coordination errors for excess stock returns, Π, real output, ϕ, and inflation, y, eventually converge to zero as follows:

$$\text{Asymptotically, } E(\Pi) = E(\phi) = E(y) = 0, \qquad \text{(T9)}$$

$$\text{if } x^* = \hat{p}_e = 0, \; \varepsilon_1 \sim N(0, \sigma_1).$$

This long run equilibrium for the economy is consistent with a money market where the expected excess real liquidity, L, and the expected excess demand for real money, θ, are both zero as follows:

$$\text{Asymptotically, } E(L) = E(\theta) = 0, \qquad \text{(T10)}$$

$$\text{if } x^* = \hat{p}_e = 0, \; \varepsilon_1 \sim N(0, \sigma_1).$$

The complex evolution of the economy converges to a simple state, where each market corrects its own errors by independent Langevin equations.

How the Fed targets interest rates determines whether the whole economy behaves rationally or not. Guesnerie and Woodford (1995) state it simply, "... An interest-rate pegging regime provides an anchor for expectations." Then it becomes easier to forecast and plan, which eventually stabilizes the stock market and the whole economy. Clearly, Wicksell's monetary policy provides the necessary guidance, such that everyone, including the central bank, can make better forecasts that reduce economic uncertainty. In fact it speeds up the smooth convergence to a stochastic steady state, where expectations eventually become more rational.

5. Conclusion

Theoretically, a simple monetary rule ideally works, if it targets inflation and nominal interest rates to both remain low. But some economists claim that useful day-to-day implementation of this goal has yet to be devised. According to Mankiw (2006):

"The sad truth is that (modern) macroeconomic research... has had only minor impact on the analysis of monetary policy... The fact that (this) research is not widely used in practical policymaking is prima facie evidence that it is of little use... "

This paper proves that the theory of monetary policy can be made more relevant, if it exploits how a complex learning economy learns from its mistakes. This search for an error correcting stationary process can be disrupted in the short run, if a central bank distorts the interest rate's normal relationship with excess stock

returns. To avoid this confusion the short-term, nominal interest rate should be pegged to eventually equal its real expectation, such that the money supply's trend be targeted to grow at a non-inflationary rate. It is easy to see how this new policy can be implemented. The nominal interest rate target can be calculated by adding inflation expectations to the expectation of the real rate. For example, as of January 26, 2009 the expectation of the annual inflation rate in the U.S. is approximately 1.1%, which is the difference between the nominal interest rate of a 30 year U.S. government bond and the real interest rate of a 30 year U.S. inflation indexed TIPS bond. Adding this projected inflation premimum to the average real interest rate, which Campbell, Lo, and MacKinlay (1997) estimate to be 1.8%, makes the nominal interest rate expectation to be almost 3%.

This means that the Fed's current federal funds target rate of less than .25% or greater than zero is biased too low. A cheap interest rate policy to stimulate the economy raises the risks of higher inflation and possibly stagflation in the long run. To control these risks the Fed eventually must raise interest rates. But it could over-react and raise rates too much, causing another boom-bust cycle. Then the Fed will again be forced into bigger bailouts of the financial sector to avoid what Fed Chairman Bernanke calls a ``chaotic unwinding'' of the whole economy (April 3, 2008, USA Today, p. B1).

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APPENDIX: NEW KEYNESIAN IMPLICATIONS

It is now possible to analyze different aspects of the evolution of a dynamic Keynesian model of a learning economy, by proving the lemmas about the existence of the IS, LM, and Phillips curves and deriving their behavior. Lemma (L1) that describes the negative slope of the IS curve's relationship between real economic output, Q, and the nominal interest rate, r^*, on the stationary locus of ϕ follows from assumption (A1) and is given by:

$$\frac{\partial r^*}{\partial Q} < 0, \text{ if } \dot{\phi} = 0, \ \phi = \ln Q - \ln Q_T, \tag{L1}$$

The positive slope of the LM curve's relationship between real output and the nominal interest rate on the stationary locus of x^* is explained in (L2) and follows from assumptions (A3) and (A5):

$$\frac{\partial r^*}{\partial Q} > 0, \text{ if } \dot{x}^* = 0, \ x^* = r^* - r_e^*. \tag{L2}$$

Also the forecast errors for inflation can cycle in complex ways. For example, under the right conditions a traditional Phillips curve exists as stated in (L3), which represents a positive trade-off between inflation and economic output on the stationary locus of y that follows from assumption (A4):

$$\frac{\partial \hat{p}}{\partial Q} > 0, \text{ if } \dot{y} = 0, \ y = \hat{p} - \hat{p}_e. \tag{L3}$$

Finally, Keynes envisioned that there is both a transaction and speculative demand for money that varies directly to the coordination error, ϕ, of the log of real output and is inversely related to the forecast error, x^*, of the nominal interest rate and excess real liquidity, L. Excess liquidity is defined as the difference between the natural logs of the real money supply and its expectation. For simplicity, assume that the expectation of the real money supply is M_T / p_e, which is the trend level of the nominal money supply, M_T, which is set by policy divided by the expectation of the price level p_e. If expectations of the supply and demand for money are equal, then Assumption (A5) can be restated as follows:

$$\theta = -l_1 x^* + l_2 \phi - L, \tag{L4}$$

$$\text{if } \ln \frac{M_T}{p_e} = -l_1 r_e^* + l_2 \ln Q_T,$$

$$\text{where } \theta = -l_1 r^* + l_2 \ln Q - \ln \frac{M}{p}, \ L = \ln \frac{M}{p} - \ln \frac{M_T}{p_e}.$$

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Table 1
A Complex Learning Economy

$$\dot{\phi} = \hat{Q} = \alpha_1 \ \Pi - \alpha_2 \ \phi, \text{ if } \dot{\hat{Q}}_T = 0, \qquad (A1)$$

such that $\phi = \ln Q - \ln Q_T$, $Z - R - r$, $\Pi = Z - z_e$.

$$\dot{z} = \dot{R} = \beta_0 - \beta_1 \ x^* \ z - \beta_2 \ z + \varepsilon_1, \text{ if } \dot{r}_e = 0, \ \dot{z}_e = 0, \qquad (A2)$$

such that $z = R - r_e$, $z_e = R_e - r_e = \dfrac{\beta_0}{\beta_2} > 0$, $\varepsilon_1 \sim N(0, \sigma_1)$

and β_2 is sufficiently large to ensure fast convergence.

$$\dot{x}^* = \dot{r}^* = \alpha_1 \pi^* + \alpha_2 \theta, \text{ if } \dot{r}_e^* = 0, r_e^* = r_e + \hat{p}_e,$$

$$(A3)$$

such that $x^* = r^* - r_e^*$, $\pi^* = \pi + y$, $\pi = z - z_e$.

$$\dot{y} = \dot{\hat{p}} = -b_1 x^* + b_2 y + b_3 \theta, \text{ if } \dot{\hat{p}}_e = 0, \qquad (A4)$$

such that $y = \hat{p} - \hat{p}_e$.

$$\theta = - l_1 r^* + l_2 \ \ln Q - \ln \frac{M}{p}, \text{ if } r^* = r + \hat{p}. \qquad (A5)$$

(Please see Table 2 for Definitions)

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TABLE 2

Let the parameters as positive constants and the variables be defined as follows:

ϕ: coordination error of the natural log of the economy's real output, Q.

Q_T: trend real output, measured by Gross Domestic Product (GDP).

R: real returns of a diversified stock portfolio, with an expectation of R_e.

Z: real excess stock returns.

z_e: market risk premium, that is the expectation of z.

Π: forecast error of excess real stock returns with respect to its expectation.

z: excess real stock returns with respect to the expectation of the real rate.

π^*: forecast error of the excess nominal stock returns, z^*.

π: forecast error of excess real stock returns, z.

ε_1: normally distributed shocks of news.

σ_1: standard deviation of ε_1.

x^*: forecast error of the nominal short-term rate, r^*.

r_e^*: expectation of the nominal rate of interest.

x: forecast error of the real short-term interest rate, r.

r_e: expectation of the real rate of interest.

y: forecast error of inflation, $\hat{p}$, compared to its expectation, $\hat{p}_e$.

θ: excess demand for real money.

M: nominal money supply, which has a trend of M_T.

p: price level.