Global Economy and Finance Journal
Volume 3. Number 2. September 2010. Pp. 44 - 60

The Simplest Model of Financial Crisis

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A Sprott three dimensional, nonlinear system of forecasting real stock returns, interest rates, and inflation can emerge from a dynamic Keynesian macro model of the economy, perturbed by noise. This is the simplest way to apply chaos theory to model financial crises. Unlike standard finance models, the proposed theory proves that monetary policy can destabilize financial markets by raising interest rates too high to correct an economy from over-expanding. This policy dilemma is the consequence of stimulating economic growth by keeping interest rates too low for too long a time. Most central banks are currently repeating this mistake, just as they did to stimulate their economies to escape the last recession These cheap money policies increase inflation expectations, accelerating stock returns that fuel an unsustainable bubble economy, causing the next crisis.

Field of Research: Sprott Nonlinear System, Monetary Policy & Financial Crisis

1. Introduction

Economists have so far failed to adequately explain the dynamics of how the economy learns from its mistakes. Muth (1961) claims that economic expectations are rational, such that forecasts are unbiased since expected errors are always zero. According to Guesnerie and Woodford (1995):

"Contrary to Muth's claim ... (that) the rational expectations hypothesis is nothing else than the extension of the rationality hypothesis to expectations, we need a theory why the rational expectations equilibrium is reached..."

The failure to predict the dynamics of how financial markets correct their mistakes explains why economists, including central banks, cannot make reliable forecasts. This is especially true, when predicting economic turning points. Then policy, based on inaccurate and skewed predictions, can make the financial markets even more risky and chaotically uncertain. These errors can precipitate a crisis, causing chaotic business cycles to emerge, such as the current Great Recession. It is now possible to meet the challenge of developing a more complete dynamic model of the evolution of a complex learning economy. This complexity is due in large part to the nonlinear feedback between the stock and money markets, creating the potential for financial instability. According to Keynes (1936):

"The latter stages of the boom are characterized by... conditions, which are unstable and cannot endure. A boom is a situation in which over-optimism triumphs... (that) in a cooler light, would be seen as excessive. "

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The resulting speculative bubble, caused by low interest rates, can be corrected when the short-term nominal interest rate is properly targeted to reduce dynamic uncertainty. But financial markets can suddenly panic, when interest rates are raised too high to curtail excessive speculation. This paper proposes to extend the dimensions of a traditional Keynesian model of the macro economy by incorporating the speculative uncertainty observed by Keynes. This macro approach is more tractable to analyze than a micro model of a representative economic agent that does not exist. Predicting the collective action of interacting heterogeneous agents is the only way to make reliable financial forecasts. So it should not be surprising that dynamic stochastic general equilibrium models, based on the behavior of an isolated representative agent, fail to address the most basic questions about explaining and predicting the economy (Colander, Howitt, Kirman, Leijonhufvud, & Mehrling, 2008).

Previously, I proved that the financial markets can behave chaotically like a Rössler nonlinear model, which predicted the Great Recession that started in late 2007, caused by high interest rates (Haley, 2009). In this paper I prove that there is a simpler way to model the chaos of financial forecasts in three dimensions and continuous time. Specifically, a Sprott nonlinear model perturbed by noise can represent the evolution of forecast errors of real stock returns, interest rates, and inflation.

Furthermore it can be proven that there exists a monetary policy, which guides the economy's search for a rational expectations equilibrium. Specifically, to promote stability the nominal interest rate should be targeted to equal its real interest rate expectation, such that the expectation of inflation is zero. This policy makes the forecast errors of stock returns behave like a Langevin error-correcting, differential equation, which smoothly and quickly converges to a normal density of errors with a bounded variance and a mean error of zero. In other words financial markets can behave rationally by making reliable and unbiased forecasts in the long run. Then everyone becomes rational, if economic predictions are based on the right monetary targets for inflation expectations and interest rates.

2. A Complex Learning Economy

The system of differential equations, summarized in Tables 1 & 2, explains how a complex economy learns to correct its forecast and coordination errors. This search evolves in continuous time with interactions among markets and not representative individuals, making the analysis more tractable of how an economy actually behaves. Once a central bank selects its monetary policy, the dynamics of economic errors implies a specific economic model or monetary regime, which predicts the actual levels of real output, real stock returns, interest rates, inflation, and money. Assume that real output for a closed economy, Q, generally mean-reverts to its trend as it dynamically adjusts varying directly to excess stock returns, Z, the difference between risky nominal stock returns, R, and the nominal short-term, risk-free interest rate, r. Assume that real output rises (falls), as the forecast error, Π, for excess nominal returns, Z, increases (decreases) with respect to its fixed expectation, z_e. This can be interpreted to mean that investment plans do not always equal planned savings except when the excess return equals its expectation. Then investment booms or busts no longer exist, since the economy converges to its trend level of output.

The following stochastic linear differential equation explains this evolution of the forecast errors, ϕ, of the natural logarithm of the economy's real output, Q, with respect to its expectation, which for simplicity is assumed to be its trend level, Q_T, that grows at a constant rate:

$$\dot{\phi} = \hat{Q} = \alpha_1 \Pi - \alpha_2 \phi, \text{ if } \dot{\hat{Q}}_T = 0, \ \dot{z}_e = 0, \qquad \text{(A1)}$$

such that $\phi = \ln Q - \ln Q_T$, $Z = R - r$, $\Pi = Z - z_e$, $\hat{Q} = \dot{Q}/Q$, $\dot{Q} = \dfrac{dQ}{dt}$.

Thus, this differential equation predicts the evolution of the deviations of output from its trend. The factor that stimulates the growth of real output is excess nominal stock returns, Z. Interestingly, Z equals excess real stock returns, Z^*, defined as the real stock return, R^*, which is the nominal return, R, minus inflation, $\hat{p}$, less the real interest rate, r^*, as follows:

$$Z = R - r = (R - \hat{p}) - (r - \hat{p}) = R^* - r^* = Z^*.$$

Furthermore, it can be easily proven that there is a negative slope of the stationary locus of (A1) or what the Keynesians call an IS curve (see Appendix).

The interaction of random noise with nonlinear feedback between the money and stock markets can increase the likelihood of abnormal behavior of stock returns that emerges during financial bubbles and panics (Engle & Lee, 1996). This complex speculative behavior is described in the following nonlinear stochastic differential equation that is perturbed by noise, ε_1, caused by exogenous random shocks of news, which are normally distributed with a zero mean and a standard deviation of σ_1. The followinig assumption specifies the dynamic mean-reverting behavior of the forecasts, π^*, of the real stock return, R^*, with respect to its fixed expectation, R_e^*. This error correcting behavior is disrupted by the nonlinear feedback of the square of the forecast error, x, of the nominal, short-term interest rate, r, with respect to its fixed expectation, r_e:

$$\dot{\pi}^* = \dot{R}^* = \beta_1 x^2 - \beta_2 \pi^* + \varepsilon_1, \text{ if } \dot{R}_e^* = 0, \qquad \text{(A2)}$$

such that $\pi^* = R^* - R_e^*$, $x = r - r_e$, $\varepsilon_1 \sim N(0, \sigma_1)$,

and β_2 is sufficiently large to ensure fast convergence.

Under the right circumstances, the expected forecast error, $E(\pi^*)$, of the real stock return, R^*, with respect to its expectation R_e^*, can quickly converge to zero. This occurs when the short-term nominal interest rate, r, equals its fixed expectation, r_e, and β_2 is a large positive number. In this case the forecast errors, π^*, of the real stock return, R^*, with respect to its expectation, R_e^*, behave like a Langevin equation, which mean-reverts such that the forecast errors, π^*, of real stock returns, become normally distributed in the long run.

Interestingly, if the forecast error of the nominal interest rate, x, is not zero, there exists positive skewness for the forecast error of the real stock return, π^*. In fact the following lemma (L1) is easily deduced from assumption (A2):

$$\frac{\partial \dot{\pi}^*}{\partial \hat{p}_e} = -2\beta_1 x > 0, \ 0 \text{ if } x = r - r_e < 0. \qquad \text{(L1)}$$

In other words if monetary policy targets the nominal interest rate, r, to be less than its expectation, r_e ($x < 0$), the creation of this cheap money stimulates rising real stock returns, π^*, as the expectation of inflation, $\hat{p}_e$, increases. These are the conditions necessary for a stock market bubble and booming economy to emerge. Eventually the bubble bursts and the economy collapses, due to mean-reversion.

The nominal short-term interest rate of risk-free government bonds increases (decreases), if the sum of the forecast error, π^*, for the real stock return plus the inflation forecast error, y, is positive (negative) and the excess demand for real money, θ, is positive (negative). Then the evolution of the forecast errors, x, of the nominal, short-term interest rate, r, with respect to its fixed expectation, r_e, is given by the following linear differential equation:

$$\dot{x} = \dot{r} = a_1 (\pi^* + y) + a_2 \theta, \text{ if } \dot{r}_e = 0, \ r_e = r_e^* + \hat{p}_e \qquad \text{(A3)}$$

such that $x = r - r_e$, $\pi^* = R^* - R_e^*$, $y = \hat{p} - \hat{p}_e$.

Interestingly, if a_1 in the above differential equation is zero, assumption (A3) resembles the standard Keynesian dynamics of the money market, depending on how excess demand for real money, θ, is specified (Ferguson & Lim, 1998). In fact θ can make this differential equation behave in a mean-reverting manner, such that there is a positive slope of what Keynesians call the LM curve (see Appendix). But financial chaos can emerge in the money market, when a_1 is positive, because the nonlinear dynamics of the stock market can affect the money market. Furthermore Rigobon and Sack (2003) have observed this interaction.

Assume that the inflation rate, $\hat{p}$, accelerates (decelerates) when the forecast errors, x, of the short-term, nominal interest rate, r, become negative (positive). Also assume that inflation increases (decreases), when there is an excess demand (supply) for real money, θ. Finally forecast errors of inflation, y, can persist, if b_2 is a small positive number. Inflation can only mean revert with the right monetary policy. The following linear differential equation models these dynamics, assuming for simplicity that the expectation of inflation, $\hat{p}_e$, is constant:

$$\dot{y} = \dot{\hat{p}} = -b_1\, x + b_2\, y + b_3\, \theta, \text{ if } \dot{\hat{p}}_e = 0, \tag{A4}$$

such that $y = \hat{p} - \hat{p}_e$, $x = r - r_e$.

This equation implies that the forecast errors for inflation can cycle in complex ways, such that variations of interest rates, inflation, and money supply can distort inflation forecasts. For example, under the right conditions about how excess demand for real money, θ, is specified, it can be shown that a traditional Phillips curve exists, such that there is generally a positive trade-off between inflation and economic output (see Appendix). In this regime, inflation can persist. Also it can be proven that a rational monetary policy exists that makes the Phillips curve vertical. Then forecast errors of inflation mean-revert and do not affect expected output in the long run, as inflation persistance disappears. This changing persistence, caused by different monetary policies, has been observed by many economists (Cogley, Primiceri & Sergent, 2010).

To complete this dynamic system of error correcting equations an assumption must be made about the behavior of the excess demand for real money, θ, that is defined as the demand for real money less than the supply of real money, M/p, both measured as natural logarithms. So assume that demand for the natural log of real money is inversely related to the short-term, nominal interest rate, r, which equals the real risk free, short-term rate of interest, r^*, plus the rate of inflation, $\hat{p}$. Also assume that real demand for money varies directly with the natural log of real output, Q. Then excess demand for money behaves as follows:

$$\theta = -\, l_1\, r + l_2 \ln Q - \ln \frac{M}{p}, \text{ if } r = r^* + \hat{p}. \tag{A5}$$

It is possible to restate the definition of the excess demand for real money as varying directly with the coordination error, ϕ, of the log of real output and varying inversely to the forecast error, x, of the nominal interest rate and excess real liquidity, L^*. Consequently, the concepts of transaction and speculative demand for money, as Keynes saw it, becomes clearer (see Appendix).

3. When Financial Chaos Erupts

The evolution of a complex economy's forecasting errors, summarized in Tables 1 & 2, is best understood by analyzing the dynamics holistically. One way to see the system's complexity is to discover how different monetary policies affect the economy's process of learning from its mistakes. By experimenting with alternative policy rules it can be proven that a monetary policy exists that efficiently speeds up the convergence to an unbiased stochastic steady state of forecasting and coordination errors. An approach that should simplify the proof of the existence of this rational expectations equilibrium is to reduce the uncertainty that distorts and disrupts the economy. One source of noise, resulting from random news, specified by ε_1 in (A2), cannot be controlled, since the variance of this noise is exogeneous and is not affected by monetary policy. But increased financial instability may be caused by the expectation of inflation or deflation that disrupts the economy, according to Meltzer (1986). This is why Taylor (1993) and other economists believe that maintaining long-term price stability reduces volatility in the economy.

The benefits of this policy become evident by assuming that the standard deviation of inflation expectations, σ_2, is zero, only if the expectation of inflation, $\hat{p}_e$, is also zero, This assumption about how the expectation of zero inflation reduces economic uncertainty is stated as follows:

$$\hat{p}_e = 0 \text{ if and only if } \sigma_2 = 0. \qquad \text{(A6)}$$

So far, the model specified by assumptions (A1) through (A6) is complete in terms of describing the evolution of economic errors. If the objective is to predict the behavior of output, real stock returns, interest rates, inflation, and money supply, a specific monetary policy is required to completely describe possible alternative monetary regimes of a complex learning economy. Knowing how the money supply behaves, based on an interest rate target, completes the dynamic system of predicting the economy's evolution. Consequently, sufficient central bank intervention makes it possible to avoid indeterminancy in the model (Cass, 1995).

It can be proven that there exists a “Keynesian” policy, which approximates how most central banks now set interest rates. It requires targeting the short-term nominal interest rate, r, to vary directly with its expectation, r_e, and the forecast error for the natural log of real economic output, ϕ. Furthermore, r varies inversely with excess real liquidity, L^*, which is the difference of the natural log of real money, $\frac{M}{p}$, and its expectation, measured by its trend, $\frac{M_T}{p_e}$. This monetary policy is mathematically specified as follows:

$$r = r_e + w_1 \phi - w_2 L^*, \quad \text{if} \quad w_1 = \frac{l_2}{l_1},\ w_2 = \frac{1}{l_1}, \tag{A7}$$

such that $\phi = \ln Q - \ln Q_T$, $L^* = \ln \frac{M}{p} - \ln \frac{M_T}{p_e}$.

This policy rule implies that when real output, Q, is greater (less) than its trend, the target interest rate should rise (fall), assuming its expectation, r_e, and excess real liquidity, L^*, are fixed. This is consistent with a traditional Keynesian policy, which constrains the economy by raising interest rates when the economy is over-expanding, $\phi > 0$, or stimulates the economy when real output is too low, $\phi < 0$, by lowering interest rates. In fact this policy is equivalent to zero excess demand for real money, θ, stated as follows:

$$\theta = 0.$$

Applying this rule to a complex learning economy given by the differential system (A1) through (A7), makes a chaotic monetary regime emerge. By substituting θ equal to zero in assumption (A3), it easily follows that:

$$\dot{x} = a_1(\pi^* + y), \text{ if } \theta = 0.$$

Clearly, after substituting again that θ equals zero in assumption (A4), it simply follows that:

$$\dot{y} = -b_1 x + b_2 y, \text{ if } \theta = 0.$$

Combining the above reduced form equations: (A2), (A3) and (A4) it follows from (A5), (A6), and (A7), that there exists a nonlinear dynamic model of financial markets unperturbed by noise from news and inflation expectations, and is given by

$$\dot{\pi}^* = \beta_1 x^2 - \beta_2 \pi^*,$$

$$\dot{x} = a_1(\pi^* + y),$$

$$\dot{y} = -b_1 x + b_2 y.$$

For simplicity assume:

$$\beta_1 = \beta_2 = a_1 = b_1 = 1,\ b_2 = 0.5. \tag{A8}$$

Therefore, after making substitutions:

$$\dot{\pi}^{*} = x^{2} - \pi^{*}, \tag{T1}$$

$$\dot{x} = (\pi^{*} + y), \tag{T2}$$

$$\dot{y} = -x + .5y. \tag{T3}$$

This three dimensional nonlinear system represents one of the simplest ways to model chaos in continuous time and is consistent with a chaotic system first devised by Sprott (1994). Since excess stock returns affect real output's growth, the following proposition about the whole economy clearly follows from assumptions (A1) through (A8):

> Financial markets can behave chaotically like a Sprott system perturbed by noise, creating unsustainable speculative bubbles, when interest rates are too high or low, implying that a Keynesian dynamic model of the macro economy cycles aperiodically. (T4)

Thus the economy continuously fails to correct its forecasting and coordination errors, because the central bank overreacts to deviations in real output and excess liquidity. Therefore as an economy recovers from recession it can overshoot its trend level of real output, since there is confusion about the economy's true direction due to the misspecified guidance provided by the central bank.

4. A Better Way to Stabilize the Stock Market

Whether expectations become rational critically depends on the stock market being efficient, such that it quickly converges to a stochastic equilibrium. Then expected excess returns of a diversified portfolio of stocks equal the market risk premium, and the probability density of excess returns is asymptotically normal with bounded variance. The stock market only avoids the financial chaos, caused by nonlinear feedback from the stock and money markets, when financial markets mean-revert, a necessary condition for rational expectations to emerge. Thus the variance, caused by the random shocks of news, is reduced asymptotically by reinforcing mean-reversion as the economy learns to correct its errors in an unbiased and more reliable manner.

In order to further stabilize financial markets it is necessary to peg the nominal interest rate, r, to equal its expectation, r_e. This new monetary policy assumption would replace the current monetary regime stated in (A7):

$$r = r_e. \tag{A7}^*$$

Now it is easily follows from assumptions (A6) and (A7)* that the interest rate peg should equal its real rate expectation, r_e^*, if price stability is the other necessary condition for reducing economic uncertainty:

$$r = r_e^* + \hat{p}_e = r_e^*, \text{ if and only if } \hat{p}_e = 0. \qquad \text{(T5)}$$

Furthermore, it can be proven that the excess demand for real money, θ, behaves as follows:

$$\theta = -\frac{a_1}{a_2}(\pi^* + y).$$

Then it follows that the complex learning economy reduces to a self-correcting monetary regime, after substituting the above values of x and θ into the appropriate error-correcting differential equations in Table 1. So a different monetary regime emerges by replacing assumption (A7) with (A7)*.

A new system of differential equations that are perturbed by noise can be derived. Specifically, after substituting assumption (A7)* into (A2), the following error correcting, Langevin equation is implied for the forecast error, π^*, of a real stock return, R^*, with respect to its expectation, R_e^*, shocked by noise, ε_1:

$$\dot{\pi}^* = -\beta_2 \pi^* + \varepsilon_1, \text{ if } x = \hat{p}_e = 0, \ \varepsilon_1 \sim N(0, \sigma_1) \qquad \text{(T6)}$$

Furthermore, the long run stochastic equilibrium of this mean-reverting differential equation can be proven to be a normal density of π^* with a zero mean and standard deviation of σ_3, which is given by:

$$\text{Asymptotically, } \pi^* \sim N(0, \sigma_3), \text{ if } x = 0, \qquad \text{(T7)}$$

such that $\lim_{t\to\infty} E(\pi^*) = 0$, $\lim_{t\to\infty} \sigma_3^2 = \sigma_1^2/2\beta_2$, $\pi^* = R^* - R_e^*$.

Eventually the long-term variance of the unbiased forecast errors of real stock returns, σ_3^2, is less than the variance of the continuous random shocks of news, σ_1^2, assuming that β_2 is relatively large (Lasota & Mackey, 1994).

Recent research tends to model the density of stock returns as not being normally distributed, especially for the short run returns (Campbell, Lo, & MacKinley, 1997). In fact actual returns exhibit excess kurtosis or fat tails as described in the attached Figure. Thus positive and negative extreme returns are more likely to occur than standard finance theory would predict. This paper proves that standard theory is correct, if central banks properly target interest rates to equal the expectation of the real rate. Then according to theorem (T7) real stock returns will behave “normally” in the long run, such that extreme returns are not likely.

Finally under certain parameter specifications it can be proven that the whole economy becomes rational. Specifically, it must be assumed that:

$$\frac{a_1}{a_2} > \frac{b_2}{b_3}. \tag{A9}$$

After making the necessary substitutions it can be shown that expected forecast errors for excess stock returns, Π, real output, ϕ, and inflation, y, eventually converge to zero as follows (Haley, 2009):

$$\text{Asymptotically, } E(\Pi) = E(\phi) = E(y) = 0, \tag{T8}$$

if $x = \hat{p}_e = 0,\ \varepsilon_1 \sim N(0, \sigma_1),\ \frac{a_1}{a_2} > \frac{b_2}{b_3}$.

The complex evolution of the economy converges to a simple state, where each market mean-reverts by evolving as independent Langevin equations.

Clearly, how a central bank targets interest rates determines whether the whole economy behaves rationally or not. Guesnerie and Woodford (1995) state it simply, “... An interest-rate pegging regime provides an anchor for expectations.” Then it becomes easier to forecast and plan, which eventually stabilizes the stock market and the whole economy. In fact it speeds up the smooth convergence to a stochastic steady state, such that expectations become rational in the long run.

5. Conclusion

This paper proves that the theory of monetary policy can be made more rational, if it exploits how a complex learning economy learns from its mistakes. This search for an error correcting stationary process can be disrupted in the short run, if a central bank distorts the interest rate's normal relationship with stock returns. To avoid this confusion the short-term, nominal interest rate should be pegged to eventually equal its real expectation of 1.8%, as estimated by Campbell, Lo and MacKinlay (1997).

Therefore the Federal Reserve's current federal funds target rate of less than .25% or greater than zero is biased too low. This cheap interest rate policy to stimulate the economy raises the risks of higher inflation and the emergence of a chaotic bubble economy in the long run. To control these risks and reduce uncertainty, the Fed eventually must raise interest rates. But it must not over-react and raise interest rates too much, contributing to another boom-bust cycle.

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Appendix: New Keynesian Implications

It is now possible to analyze different aspects of the evolution of a dynamic Keynesian model of an economy that learns from its mistakes. This requires proving the lemmas about the existence and behavior of the IS, LM, and Phillips curves. Lemma (L2) describes the negative slope of the IS curve's relationship between real economic output, Q, and the nominal interest rate, r, on the stationary locus of output errors, ϕ, that follows from assumption (A1) and is given by:

$$\frac{\partial r}{\partial Q} < 0, \text{ if } \dot{\phi} = 0, \ \phi = \ln Q - \ln Q_T , \tag{L2}$$

The positive slope can be derived for the LM curve's relationship between real output, Q, and the nominal interest rate, r, on the stationary locus of the forecast errors of interest rates, x. It is explained in (L3) and follows from assumptions (A3) and (A5):

$$\frac{\partial r}{\partial Q} > 0 , \text{ if } \dot{x} = 0 , \ x = r - r_e \tag{L3}$$

Also the forecast errors for inflation can cycle in complex ways. For example, a traditional Phillips curve exists as stated in (L4), which represents a positive trade-off between inflation, $\hat{p}$, and economic output, Q, on the stationary locus of inflation forecast errors, y, that follows from assumption (A4), assuming certain parameter specifications:

$$\frac{\partial \hat{p}}{\partial Q} > 0 , \text{ if } \dot{y} = 0 , \ y = \hat{p} - \hat{p}_e , \ b_2 < (b_1 + b_3 \ l_1). \tag{L4}$$

This positive trade-off exists if inflation rates persist, making b_2 a relatively small positive number.

Finally, Keynes envisioned that there is both a transaction and speculative demand for money such that the money supply is not always spent, creating excess liquidity or the "hoarding" of money. Assume excess liquidity is defined as the difference between the natural logs of the real money supply and its expectation. For simplicity, assume that the expectation of the real money supply is M_T / p_e, which is the trend level of the nominal money supply, M_T, which is set by policy and divided by the expectation of the price level p_e. Furthermore it can be assumed that the log of the trend real money supply equals the log of the demand for money that varies inversely with the expectation of the nominal interest rate, r_e, which equals the expectation of the real risk free interest rate, r_e^*, plus the expectation of inflation, $\hat{p}_e$. And assume that the expectation of real money supply, varies directly with the log of the expectation of the economy's real output, Q_T, measured by its trend, as follows:

$$\ln \frac{M_T}{p_e} = -l_1 r_e + l_2 \ln Q_T \text{, if } r_e = r_e^* + \hat{p}_e .$$

If the expectations of the supply and demand for money are equal, (A5) can be restated as follows to explain how the transaction and speculative uses of money may result from coordination errors:

$$\theta = - l_1 x + l_2 \phi - L^* , \qquad \text{(L5)}$$

$$\text{if } \ln \frac{M_T}{p_e} = - l_1 r_e + l_2 \ln Q_T ,$$

where $\theta = -l_1 r + l_2 \ln Q - \ln \frac{M}{p}$, $L^* = \ln \frac{M}{p} - \ln \frac{M_T}{p_e}$.

It is also possible to target inflation expectations by targeting the money supply in the long run. Thus the growth rate of the trend money supply, $\hat{M}_T$, varies directly with the growth rate of the economy's trend real output of the economy, $\hat{Q}_T$. As long as the expectations of inflation, $\hat{p}_e$, and the real interest rate, r_e^*, are fixed, noninflationary growth rate of the trend money supply should be targeted to be:

$$\hat{M}_T = l_2 \hat{Q}_T \text{, if and only if } \hat{p}_e = 0 ,\ \dot{r}_e = 0 . \qquad \text{(L6)}$$

If l_2 is approximately one, the expectation of price stability requires that the average growth of money should be in an estimated range of 3-3.5%, which approximately equals the trend growth of real economic output in the United States.

Table 1

A Complex Learning Economy

$$\dot{\phi} = \hat{Q} = \alpha_1 \Pi - \alpha_2 \phi, \text{ if } \dot{\hat{Q}}_T = 0, \ \dot{z}_e = 0, \quad \text{(A1)}$$

such that $\phi = \ln Q - \ln Q_T$, $Z = R - r$, $\Pi = Z - z_e$.

$$\dot{\pi}^* = \dot{R}^* = \beta_1 x^2 - \beta_2 \pi^* + \varepsilon_1, \text{ if } \dot{R}_e^* = 0, \quad \text{(A2)}$$

such that $\pi^* = R^* - R_e^*$, $x = r - r_e$, $\varepsilon_1 \sim N(0, \sigma_1)$

and β_2 is sufficiently large to ensure fast convergence.

$$\dot{x} = \dot{r} = a_1 (\pi^* + \mathrm{y}) + a_2 \theta, \text{ if } \dot{r}_e = 0, \ r_e = r_e^* + \hat{p}_e \quad \text{(A3)}$$

such that $y = \hat{p} - \hat{p}_e$.

$$\dot{y} = \dot{\hat{p}} = -b_1 x + b_2 y + b_3 \theta \quad \text{(A4)}$$

$$\theta = -l_1 r + l_2 \ln Q - \ln \frac{M}{p}, \text{ if } r = r^* + \hat{p}. \quad \text{(A5)}$$

(Please see Table 2 for Definitions)

Table 2

Definitions

Let the fixed positive parameters and the variables be defined as follows:

ϕ: forecast error of the natural log of real output, Q, with respect to its trend.

Q_T : trend real output of the economy.

R : nominal risky returns of a diversified stock portfolio, with an expectation of R_e .

r : nominal interest rate of a risk free bond.

Z : excess nominal stock returns.

z_e : market risk premium, that is the expectation of Z .

Π : forecast error of excess nominal stock returns with respect to its expectation.

π^* : forecast error of a real stock return, R^*, with respect to its expectation, R_e^* .

ε_1 : normally distributed random shocks of news.

σ_1 : standard deviation of ε_1 .

r_e : expectation of the nominal rate of interest.

x : forecast error of the nominal rate of interest, r, with respect to its expectation.

r_e^* : expectation of the real rate of interest.

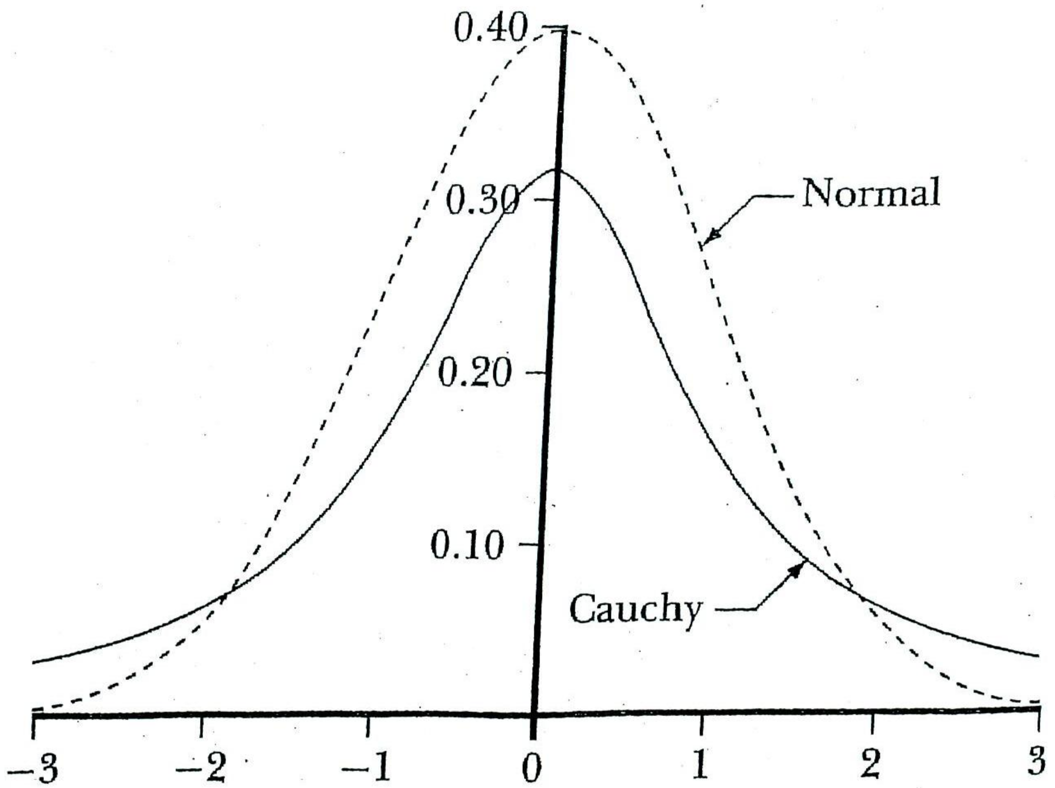
y : forecast error of inflation, $\hat{p}$, with respect to its expectation, $\hat{p}_e$.

θ : excess demand for real money.

M : nominal money supply, which has a trend of M_T .

p : price level.

Figure: Density of Forecast Errors



Comparison of Stable and Normal Density Functions