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Profit Optimization with Post Optimality Analysis Using Linear Programming

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Industrial competition pressures management to optimize operational variables, such as levels of product production. The product-mix problem is an example of situation where a multiproduct firm seeks the optimal product mix. This problem is usually formulated as a Linear Programming (LP) problem, and defines the optimal operational points for all products (decision variables). In this paper we show that the product-mix problem can be used efficiently not only to determine the optimal operational points but also to provide information on how those optimal points could be further increased through changing the constraints of the optimization problem. This additional information could be used to enhance production by informing expansion plans in which management identifies and takes advantage of the capacity of under-utilized constraints and uses them to expand the capacity of over-utilized or limiting constraints. This expansion will utilize units more efficiently and will increase the overall operational profit.

Keywords: Linear programming (LP), Sensitivity analysis (SA), Post optimally analysis (POA), Positive Sensitivity analysis (PSA), Lagrange Multiplier method.

1. Introduction

Industries all over the world are continuously faced with shortages of production resources which result in low capacity utilization and consequently low outputs. firm managers are always seeking for the right decisions so as to meet their objectives which mainly revolve on how best to increase profit (Ezema and Amakom). Many operations management decisions involve trying to make the most effective use of an organization's resources. Resources typically include machinery, labour, time, and raw materials. These resources may be used to produce products (such as machines, furniture, food, or clothing) or services (such as airline schedules, advertising policies, or investment decisions).

Many applications in business and economics involve a process called optimization, in which we are required to find the minimum cost, the maximum profit, or the minimum use of resources. Since profits are the difference between total revenues (price of one unit multiplied by the quantity sold, assuming all units are identical) and total costs,

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one can maximize profits by increasing his revenues or diminishing his costs. There are limits to how much somebody can reduce costs. A mathematical optimization model consists of an objective function and a set of constraints expressed in the form of a system of equations or inequalities. The objective function is a measure of effectiveness, often the cost or the profit. The model also includes decision variables and parameters. Optimization models include the following components (Taylor 1999).

(1) Decision variables - mathematical symbols representing levels of activity of a firm.
(2) Objective function - a linear mathematical relationship describing an objective of the firm, in terms of decision variables, that is maximized or minimized.
(3) Constraints - restrictions placed on the firm by the operating environment stated in linear relationships of the decision variables.
(4) Parameters - numerical coefficients and constants used in the objective function and constraint equations.

1.1 Linear Programming (LP)

In operation research technology, programming means the use of optimizing techniques, and linear refers to the relationship between variable. Hence, linear programming models arise from use or allocation of resources such as, materials, machines, capital, etc so as to optimize cost and optimize profit. Linear programming gives a reliable solution (it warrantees the global optimal solution) to the manager. The merits of linear programming are nowadays well established and linear programming is widely accepted as a useful tool in Operations Research and Management Science. A large number of companies are using this way of modelling to solve various kinds of practical problems. Linear programming refers to the fact that both the objective function to be optimized and all the constraints are linear in terms of the decision variables.

Linear programming provides an excellent opportunity to introduce the idea of "what-if" analysis, due to the powerful tools for post-optimality analysis developed for the linear programming model. Thus, the optimization effect is the objective of setting and solving the required linear programming models. Linear programming finds many uses in the business and industry, where a decision maker may want to utilize limited available resources in the best possible manner. One of the most valuable things about linear programming is that it is easily applicable to real life and provides various methods of solving such problems (Luenberger and Ye 2008).

1.1.1 Application of Linear Programming

In mathematics the process of finding an extreme value (maximum or minimum) of a quantity (called objective function) is known as optimization problem. Also a constraint is a condition that a solution to an optimization problem is required by the problem itself to satisfy. There are two types of constraints: equality constraints and inequality constraints. If an inequality constraint holds with equality at the optimal point, the constraint is said to be binding, as the point cannot be varied in the direction of the constraint even though doing so would improve the value of the objective function. If an inequality constraint holds as a strict inequality at the optimal point (that is, does not hold with equality), the constraint is said to be non-binding, as the point could be varied in the direction of the constraint, although it would not be optimal to do so. If a constraint is non-binding, the optimization problem would have the same solution even in the absence of that constraint. The set of candidate solutions that satisfy all constraints is called the feasible set.

In the manufacturing sectors, there are a number of activities to be performed. Good production planning, e.g., effective product mix, is essential to successful production-based business. Efficient performing of each activity needs different optimizations. These optimizations try to solve the problem of how to combine the activities and resources in order to have a maximum overall efficiency. In other word, these optimization problems try to answer the following questions in more systematic:

- What products to produce?
- How much to produce?
- What to do with products in excess of demand?
- How to increase utilization of available raw material?

The concept of hierarchical production planning was first developed by (Hax and Meal, 1975) and subsequently used by (Bitran, Haas et al. 1981) who proposed three aggregation levels: a) items the lowest aggregation level corresponding to end products delivered to the customers; b) families groups of items pertaining to a same product type and sharing similar setups; c) product types groups of families having similar cost structures, manufacturing processes and seasonalities. The top decision level in a hierarchical production planning process focuses on product type decisions. At this level, product mix, inventories, and manufacturing strategies as well as hiring and layoff decisions are reached in each planning period. The literature on product mix optimization includes a variety of models but very few consider joint resource/product mix decisions. Generally heuristic or meta-heuristic approaches are used to find solutions for product mix problems.

The company would like to determine how many units of each product it should produce which is called production scheduling. Solving a production scheduling problem allows the production manager to set an efficient, low cost production schedule for a product over several production periods. Basically, the problem resembles the common product-mix model for each period in the future. Production levels must allow the firm to meet demand for its product within manpower and inventory limitations. One way is selling under-utilized resource and using them for extending other extendable resources. In these circumstances Linear Programming can be viewed as an analytical process and a holistic decision support tool that offers many advantages to managers.

1.2 Sensitivity Analysis

Operation managers are usually interested in more than the optimal solution to an LP problem. In addition to knowing the value of each decision variable and the value of the objective function, they want to know how sensitive these answers are to input parameter changes. If the LP solution produces a sensible answer and we are confident that the model is a valid representation of the real life situation under investigation, we may then proceed to analyse the solution output in detail. For example what happens if right-hand-side values of the constraints change? Due to the fact that solutions are based on the assumption that input parameters are constant, the subject of sensitivity analysis comes into play.

Sensitivity analysis, or post optimality analysis, is the study of how sensitive solutions are at the impact of parameter changes. Sensitivity analysis is concerned with understanding how changes in the model input data's influence the outputs. The results of sensitivity analysis establish upper and lower bounds for input parameter values within which they can vary without causing violent changes in the current optimal solution. A broad range of measures have been advocated in the literature to quantify and depict the sensitivity of a model's output to change in its inputs. Other names for such activities are stability analysis, what-if analysis, scenario modeling, ranging analysis, specificity analysis, uncertainty analysis, computational and numerical instability, functional instability, tolerance analysis, post-optimality analysis, allowable increases and decreases, and many other similar phrases that reflect the importunateness of this stage of the modeling. This method has been introduced in numerous studies so far and has been used in many linear programming problems. However, in case of deviancy, it may yield imperfect information due to alternative optimal bases (Evans and Baker, 1982; Knolmayer, 1984; Jansen et al., 1997). However, there are some disadvantages in the application of sensitivity analysis.

(Yang 1990) defined two types of sensitivity analysis based on (Sung and Park 1988) definition. The first type of sensitivity analysis is defined to find the characteristic region within which an optimal basis still remains optimal for a perturbed problem. The second type, called PSA, is defined to find the characteristic region within which variables having a zero and having a positive value in an optimal solution remain zero and positive in the perturbed problem, respectively. Positive sensitivity analysis (PSA) is a sensitivity analysis method for linear programming problem that finds the range of deviancy within which positive value components of a given optimal solution remains positive (Park, Kim et al. 2004). There are some cases where PSA is useful. For example, when the supply of a certain material is changed, we need to determine the optimal output of each product under the current policy. In this circumstance PSA can be used to find the amount of the change as long as the constraint is satisfied.

1.2.1 Application of Sensitivity Analysis

In constrained optimization in economics, the shadow price is the instantaneous change per unit of the constraint in the objective value of the optimal solution of an optimization problem obtained by relaxing the constraint. In other words, it is the marginal utility of relaxing the constraint, or, equivalently, the marginal cost of strengthening the constraint. In a business application, a shadow price is the maximum price that management is willing to pay for an extra unit of a given limited resource. The value of the shadow price can provide decision-makers with insights into problems. More formally, the shadow price is the value of the Lagrange multiplier at the optimal solution, which means that it is the infinitesimal change in the objective function arising from an infinitesimal change in the constraint.

If you know accurately the shadow price of a resource, it tells you how hard to work to get more of it. This follows from the fact that at the optimal solution the gradient of the objective function is a linear combination of the constraint function gradients with the weights equal to the Lagrange multipliers. In all cases the slack activity multiplied by the shadow price will be equal to zero implying that either or both of these values must be zero. This makes intuitive sense in that it cannot benefit the objective function to increase the availability of a resource that is already under-utilised. On the other hand, if a resource is fully utilised, it is clear that changes in availability are likely to affect the optimal solution (Boyd and Vandenberghe 2004). There are a number of questions that could be asked concerning the sensitivity of an optimal solution to changes in the data. Every commercial linear programming system provides this elementary sensitivity analysis, since the calculations are easy to perform using the tableau associated with an optimal solution.

In this paper we consider the problem of estimating the variation of the objective function resulting from changes in right-hand-side values of constraints and its impact on the optimal solution. The rest of the paper is organized as follow: The next section provides background about the application and usefulness of linear programing in economic development. In Section 3 we provide the methodology of solving this optimization problem using Lagrange method and present a case study based on the discussed methodology. In the case study presented in section 3.2 using the framework of linear programming we investigate how changes in the parameters of a linear programming problem affect its optimal solution.

2. Literature Review

Resource Optimization using linear algebra involving systems of linear equations are used to solve financial decision-making problems, especially after the proliferation of user-friendly software, which facilitate easy implementation of mathematical algorithms. Linear Programming (Charnes, Cooper et al. 1963), an extensively used optimization technique, determines the values of decision variables that optimize a single objective such as profit maximization or cost minimization, subject to constraints. There are diverse opinions on the applicability of the linear programming technique to different management decision-making processes. These opinions developed over a long period of time following continuous improvement on the application of the technique in solving practical business problems.

Most literature in economic development supports the view that linear programming is a practical tool of analysis in allocating resources to their optimal use and is of vital importance to the economies of underdeveloped countries. Tracing the history of linear programming method, it is a mathematical device developed by the mathematician, George Dantzig, in 1947 for planning the diversified activities of the U.S. Air Force connected with the problem of supplies to the Force. (Dantzig 1951) has been implemented in a large variety of codes which are successfully used in practice. Not long after the publication of Dantzig's primal simplex method its dual version, developed by(Lemke 1954). In 1984, Karmarker proposed his projective algorithm for linear programming, which he showed to be a polynomial time algorithm. This gave the impulse to an enormous amount of research on interior point methods. The first polynomial algorithm for systems of linear equations was given by (Khachiyan, Kozlov et al. 1979). Afterwards, Dantzig suggested this approach for solving business and industrial problems. He also developed a powerful mathematical tool known as “simplex method” to solve linear programming problems (Momoh, El-Hawary et al. 1999).

(Sargeaunt 1965) groups, what he calls, the principal types of application of linear programming models under three headings, namely: blending and mix determination problems, planning and scheduling problems and distribution cost problems. He also notes other types of applications: for instance to plant location decisions, to personnel allocation problems, and to the analysis of a multi-plant production system to determine whether or not certain plants should be shut down as a result of high cost of production. (Emory, Niland et al. 1968) have listed similar types of problems to which linear programming is applied. They include: the blending of gasoline stocks, the formulations of various chemical products, the mixing of fertilizer, the manufacture of cement and the charging of electric furnaces. On the other hand, Hillier and Lieberman (2001) assert that although allocating resources to activities is the most common type of application, linear programming has numerous other important applications as well. In fact any problem whose mathematical model fits the very general format for the linear programming model is a linear programming problem.

Moreover, (Bernard et al. 2009) have recently questioned the relevant level of analysis for the estimation of a production function, by documenting the pervasiveness of multi-product firms in the US manufacturing sector. (Cole, Wang et al. 2009) proposed an immune mechanism algorithm approach to maximize throughput in a capacity-constrained resource. (Hasuike and Ishii 2009) investigated several production planning problems in which product-mix decisions under random and fuzzy conditions and proposed models for them. Zhang and Tseng (2009) observed that manufacturing in general is moving towards higher product mix and lower volume and incorporated customer flexibility into a proposed product mix flexibility model. (Tsoulos and Vasant 2009) proposed an evolutionary optimization technique to solve the product mix selection problem. (Rezaie, Nazari-Shirkouhi et al. 2009) proposed a meta-heuristic algorithm which they called the Imperialist Competitive Algorithm to solve the integrated product mix-outsourcing optimization problem. However the literatures on product mix optimization includes a variety of models but very few of them consider joint resource/product mix problems.

Most of the packages available for solving linear programming do not only solve the linear programming problem but also provide the information on the sensitivity of the solution to certain changes in the data. This is referred as sensitivity analysis (SA) or post optimality analysis (POA). Sensitivity analysis is a collection of post-solution activities to study and determine how sensitive the solution is to changes in the assumptions. The method of sensitivity analysis in simplex method is well developed on the foundation of optimal basis. This method has been introduced in numerous papers and textbooks so far (Dantzig)1963; Gal, 1979). This information can be of tremendous importance in practice, where parameter values may be estimates (Dahiya and Verma 2007). (Dahiya and Verma, 2005) studied sensitivity analysis using optimal bases for the linear programming problem with bounded variables. However, since sensitivity analysis using an optimal basis cannot be applied to an optimal non basic solution, other methods for sensitivity analysis have been suggested.

Yang (1990) introduced positive sensitivity analysis (PSA) for optimal solutions including optimal non basic solutions based on Sung and Park's (1988) definition for the linear programming problem with non-negative decision variables. Positive sensitivity analysis is defined to find the characteristic region within which variables having a zero and having a positive value in an optimal solution remain zero and positive in the perturbed problem, respectively. (Adler and Monteiro 1992) developed a method of parametric analysis on the right-hand side by introducing the optimal partition. (Monteiro and Mehrotra 1996) presented a parametric analysis by generalizing Adler and Monteiro's method, and (Greenberg 2000) developed a method of sensitivity analysis using the optimal partition when cost coefficients and right-hand sides change simultaneously. To use Yang's and Adler and Monteiro's method, we need an optimal solution or the optimal partition, which requires additional computation for interior-point methods. (Boyd and Vandenberghe 2004) studied PSA in linear programming that was useful for establishing the relationship between PSA and sensitivity analysis using optimal bases and using optimal partition. This motivated us to carryon this study for the case when decision variables are bounded.

3. The Methodology and Model

In this paper we use linear programming in which the problem is presented in a form of a linear objective function with linear constraints. To solve this optimization problem we apply Lagrange method. Solving a linear programming problem is straightforward and we just bring its solution here. We denote the objective function to be maximized by$\boldsymbol{f}(\boldsymbol{x}_\mathbf{i})$, and the inequality constraints by: $g_\mathrm{j}(x_\mathrm{i}) \leq 0$, where($\mathrm{i} = 1, \dots, \mathrm{m}$ and $\mathrm{j} = 1, \dots, \mathrm{l}$).

$$\boldsymbol{\ell}\left(\boldsymbol{x}_i^*, \boldsymbol{\mu}_j^*\right) = \boldsymbol{f}(\boldsymbol{x}_\mathrm{i}) + \boldsymbol{g}_\mathrm{j}(\boldsymbol{x}_\mathrm{i}) \qquad (\mathbf{1})$$

Suppose that the objective function$\mathbf{f} : \mathbf{R}^\mathbf{n} \to \mathbf{R}$ and the constraint functions $\mathbf{g_j} : \mathbf{R}^\mathbf{n} \to \mathbf{R}$ are continuously differentiable at a point x_i^*. If x_i^* is a local minimum, then there exist coefficients $\boldsymbol{\mu}_\mathbf{j}$ $(\boldsymbol{j} = \mathbf{1}, \dots, \boldsymbol{l})$, called Lagrange multipliers, such that:

$$\nabla \boldsymbol{f}(\boldsymbol{x}_i^*) + \sum_{j=1}^{m} \boldsymbol{\mu}_\mathrm{j} \nabla \boldsymbol{g}_\mathrm{j}(\boldsymbol{x}_i^*) = \mathbf{0} \qquad (\mathbf{2})$$

In constraint optimization problems, $\boldsymbol{\mu}_j^*$is called shadow price and is a measure of sensitivity of objective function to the change in the **"$\boldsymbol{j^{th}}$"** constraint. Each constraint in an optimization problem has a shadow price or dual variable. Solving equation (2) together with considering the dual feasibility and complementary slackness conditions gives the optimal point $\boldsymbol{x}_i^*$ and shadow prices $\boldsymbol{\mu}_\mathbf{j}{}^\mathrm{i}$.

Primal feasibility: $\boldsymbol{g}_\mathrm{j}(\boldsymbol{x}_i^*) \leq \mathbf{0}$, **for all** $\boldsymbol{j} = \mathbf{1}, \dots, \boldsymbol{l}$

Dual feasibility: $\boldsymbol{\mu}_\mathrm{j}$ **≥0 for all** $\boldsymbol{j} = \mathbf{1}, \dots, \boldsymbol{l}$

Complementary slackness: $\boldsymbol{\mu}_\mathrm{j} \boldsymbol{g}_\mathrm{j}(\boldsymbol{x}_i^*) = 0$ **for all** $\boldsymbol{i} = \mathbf{1}, \dots, \boldsymbol{m}$ and $\boldsymbol{j} = \mathbf{1}, \dots, \boldsymbol{l}$

3.1 Profit Optimization Using Analytic Post Optimality Method

There are two approaches to determining just how sensitive an optimal solution is to changes. The first is simply a trial-and-error approach. This approach usually involves resolving the entire problem, preferably by computer, each time one input data item or parameter is changed. It can take a long time to test a series of possible changes in this way. The other method is the analytic post optimality method. After an LP problem has been solved, we determine a range of changes in the right-hand-side values of the constraints that will increase the optimal profit. The sensitivity range for a right-hand-side value is the range of values over which the quantity's value can change without changing the solution variable mix. If the right-hand side of a constraint (resource available) is changed, the feasible region will change (unless the constraint is redundant), and often the optimal solution will change. If you already have lots of the resource, and you do not expect to use it up, the shadow price must be zero.

3.2 Case Study

In this case study we Consider a Company, manufacturer of two products. The problem is to determine the optimal level of output for two products x_1 and x_2. Products consist of three types of raw materials in the absence of having certain and limit amount of these resources. Primarily the goal is to maximize the total profit from the production of the two products with the constraint on the amount of raw materials.

$$\textbf{Max } \pi = 100x_1 + 60x_2 \qquad \textbf{(3)}$$

The objective function will be described production process has several resource constraints. First constraint, consider production of x_1(units of product 1) which requires 5 units of raw material 1 and Similarly, production of x_2(units of product 2) which requires 2 units of the same raw material. The sum of these two quantities of raw materials must be less than or equal to the quantity available, which is 40 units. This relationship can be expressed as:

$$5x_1 + 2x_2 \leq 40 \qquad \textbf{(4)}$$

Similarly assume that there are two other raw material limitations for producing of Product **1**and Product **2**:

$$20x_1 + 40x_2 \leq 400 \qquad \textbf{(5)}$$

$$10x_1 + 50x_2 \leq 300 \qquad \textbf{(6)}$$

Solving this optimization problem gives the optimal value of $x_1^* = 6$ and $x_2^* = 4$ unit with the profit of π = 840 ($). Table (1) shows the results of the basic optimization problem. The feasible area (for solution) together with the optimal point is shown in figure (1). This figure shows that second constraint ($20x_1 + 40x_2 \leq 400$) is inactive and has no effect on determining of the optimal point. Mathematically it means that shadow price of this constraint is equal to zero. On the other hand, the bigger values of shadow price shows the stronger effects that changing one unit of that constraint can have on the overall objective function. In other word, if we can expand the constraint related to the highest value of shadow price, we will increase the objective function (profit) more efficiently.

Table 1: The results of basic mixed product problem

Shadow Prices	$\mu_1 = 19.13$	$\mu_2 = 0$	$\mu_3 = 0.43$
x^*	$x_1^* = 6.09$	$x_2^* = 4.78$	
Max π	π = 895.65 ($)		

In this example we have assumed that first equation is not expandable and we want to expand 3^{rd} constraint with releasing unused capacity of second constraint. This can be written in the following optimization problem.

$$\text{Max} \quad \pi = 100x_1 + 60x_2 \qquad (7)$$

$$\text{S.t.} \quad 5x_1 + 2x_2 \leq 40$$
$$20x_1 + 40x_2 \leq 400 - K$$
$$10x_1 + 50x_2 \leq 300 + K \times m$$

This optimization is similar to the basic product-mix problem (3) with the difference that we are optimizing it in different steps K for expandable constraints. In this process we release K unit of unused row material in second constraint so we can expand third constraint with $K \times m$ steps. We have assumed that the worthiness for K unit of row material in second constraint is equal to the worthiness of $K \times m$ unit for row material in the third constraint. We have solved optimization problem (7) for $K = 50$ with three different values of $m = 0.5, 1,$ and 2. Figure 1 shows the profit of optimization for these cases. For the case that $m = 1$, new optimal point is shown in figure 2. Indeed new optimal point is the point that the value of shadow price is equal for both of constraints. This means that any change in the value of K, activates one of constraints and simultaneously the other constraint becomes inactive.

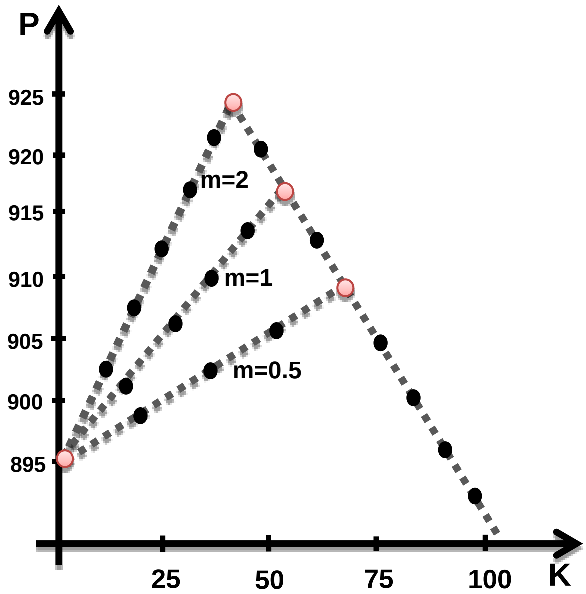

Figure 1: Optimal value of product-mix problem for different values of "k" and "m"

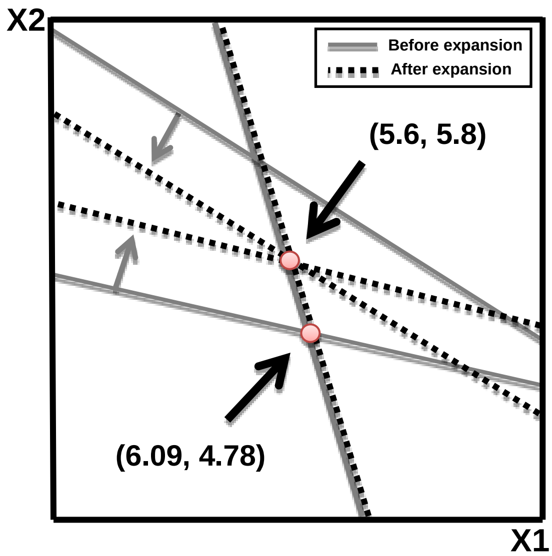

Figure 2: Optimal value of product-mix problem after and before expansion

4. The Findings

In many under developing countries, imbalance usage of resources has changed the optimal operating point for different industries. Hence in a simple manufacturing unit for long-term planning it is important to know the bounds within which each available resource, (e.g., machine hours) can vary. With this information, management can improve the current optimal operating point by proper diffusion of resources. This improvement indeed is because of what we suggest in expanding the binding constraint of basic product-mix problem. The more important finding in this study is that; we did this by any external investigation or other resource provision. Our post optimality analysis in section 3.2 shows that how expanding the constraint with high shadow prices, has tangible effect on the overall profit.

The findings of this paper indicate that, managers in the manufacturing sector can increase the overall profit by selling under-utilized resource and using them for extending other extendable resources. This result shows that, efficient utilization of resource would have a detrimental effect on the objective function and would increases the overall operational profit. The problem formulation is also based on the linear programming that should be solved for different steps. In this case we found different Post Optimal values of product-mix problem in different values of "k" and "m". As it's shown in Figure 2 there is a right-hand side range over which; tightening constraint 2 by the amount of "k" unit and Relaxation constraint 2 by the amount of "km" unit, would benefit the objective function value. We can see that these Post Optimal values are at the impact of "m" values. This approach is extendable to many other types of expandable constraints in the manufacturing units.

5. Summary and Conclusions

The growth of industries puts pressure on the management to find the optimal operational points such as controlling levels of production. Product-mix problem is an example of situation where a multiproduct firm should determine the optimal product mix. This problem usually is formulated as a Linear Programming (LP) problem and defines the optimal operational points for all products (decision variables). In linear programming, all model parameters are assumed to be constant; but in real life situations, the decision environment is always dynamic. Therefore, it is important for the management to know how profit would be affected by an increase or decrease in the resource level, by a change in the technological process, and by a change in the cost of raw materials. Such an investigation is known as sensitivity analysis or post-optimality analysis. Sensitivity analysis or post optimal analysis focuses on the induced partition of primal-optimal solutions.

Most of the packages available for solving linear programming do not only solve the linear programming problem but also provide the option to ask for information on the sensitivity of the solution to certain changes in the data. In section 3.2 using the framework of linear programming we investigate how changes in the parameters of a linear programming problem affect its optimal solution. When right-hand side value of a constraint is changed, the range of sensitivity analysis finds the interval where the induced partition of a given optimal solution remains the induced partition of some optimal solution to the perturbed problem. This information can be of tremendous importance in practice, where parameter values may be estimates.

We use a post optimality analysis of the established profit maximization model that would be attempted to help the management in adjusting its decisions in the face of increases or decreases in resource (availability of raw materials). Also we showed that the product-mix problem can be used efficiently to define not only the operational points but also it gives strong suggestions to change the optimal operating points. These suggestions are expansion plans in which, management tries to release the capacity of under-utilized constraints and use them to expand the over-utilized constraint.

Endnote

[i] There are a lot of free software which that can solve the LP problem efficiently. In this paper we use the Linprog command of MATLAB which gives both x_i^* and μ_j.

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